

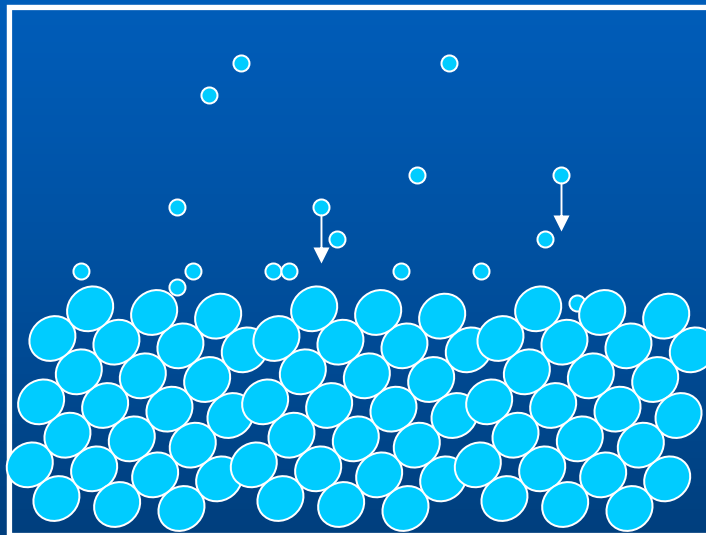
FRACTAL DIMENSIONS AND SETTLING PROPERTIES OF CHEMICAL COAGULATION FLOCS

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UNIVERSITY
OF MANITOBA

Gravity Settling



Individual flocs

Floc blanket –
solid/liquid
interface zone
settling

Settling of Individual Floccs

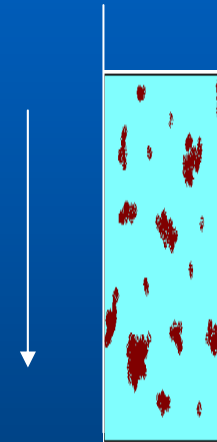
Stokes' Law

$$V_t^2 = \frac{4g(\rho_F - \rho_w)d}{3\Omega C_D \rho_w}$$

Density

Drag

Size



**Inaccurate Floc Settling
Velocity Predictions**

Settling of Floc Blanket

$$V_{zs} = k \exp(-nC)$$

C – sludge concentration

K.n. - empirical coefficients specific for each sludge

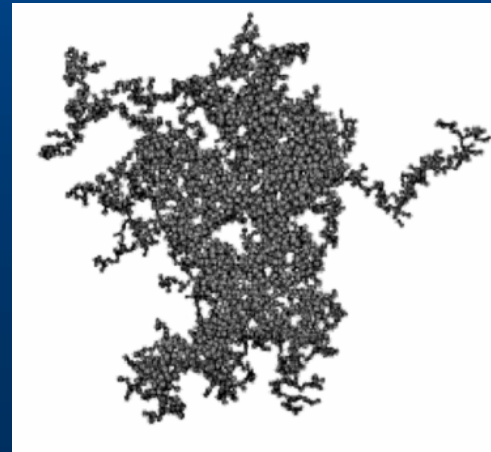
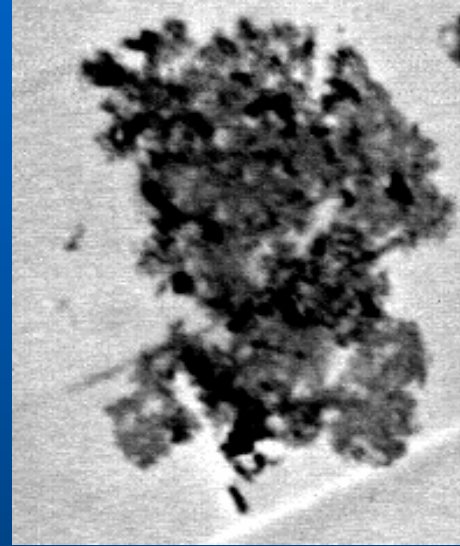
Parameter	Alum Coagulation Sludge	Activated Sludge
k_1 (m/h)	2.57 (Zheng and Bagley, 1999)	13 (Ekama et al., 1997)
v_t (m/h)	2.52	13.68
k_2 (kg/m ⁴ /h)	$3.61 * 10^5$ (Zheng and Bagley, 1999)	$3.62 * 10^4$ (Zheng and Bagley, 1999)
$(\rho_s - \rho_w) u / k$ (kg/m ⁴ /h)	$2.25 * 10^5$	$9.95 * 10^4$
n_2 (m ³ /kg)	0.5 (Zheng and Bagley, 1999)	0.64 (Zheng and Bagley, 1999)
ρ_s^{-1} (m ³ /kg)	0.4 (Gorczyca, 1991)	0.71 (Li and Ganczarzyk, 1987)

Objectives of this study

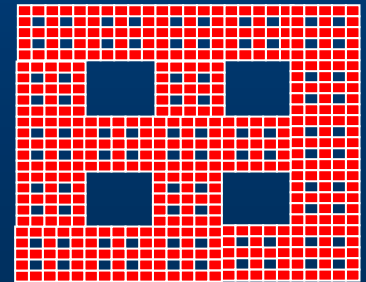
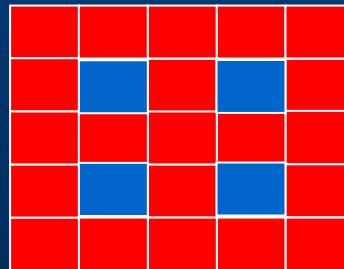
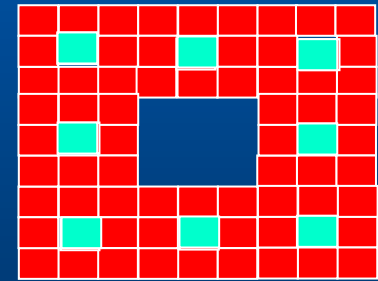
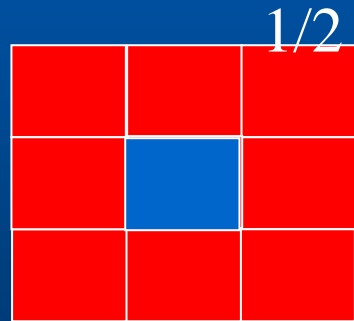
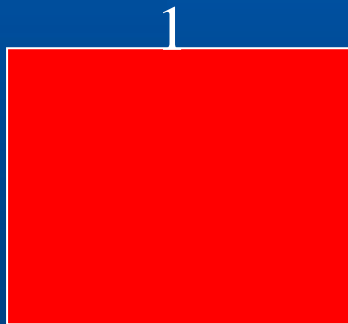
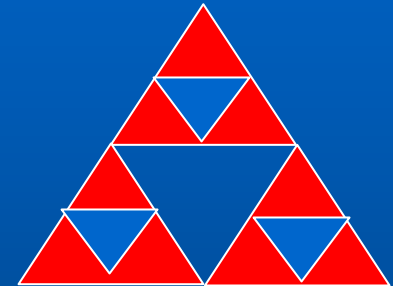
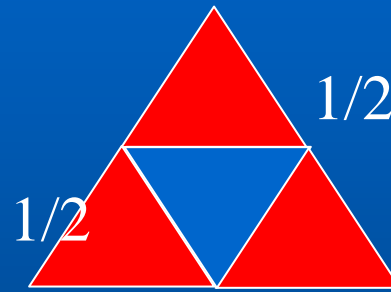
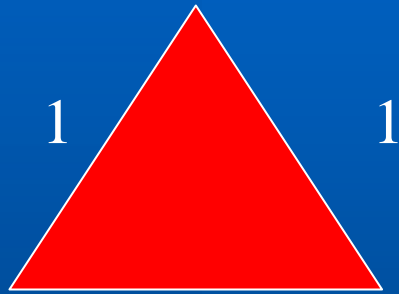
Analyze Properties of Floccs

Model Settling of Floccs

Flocs are Fractals

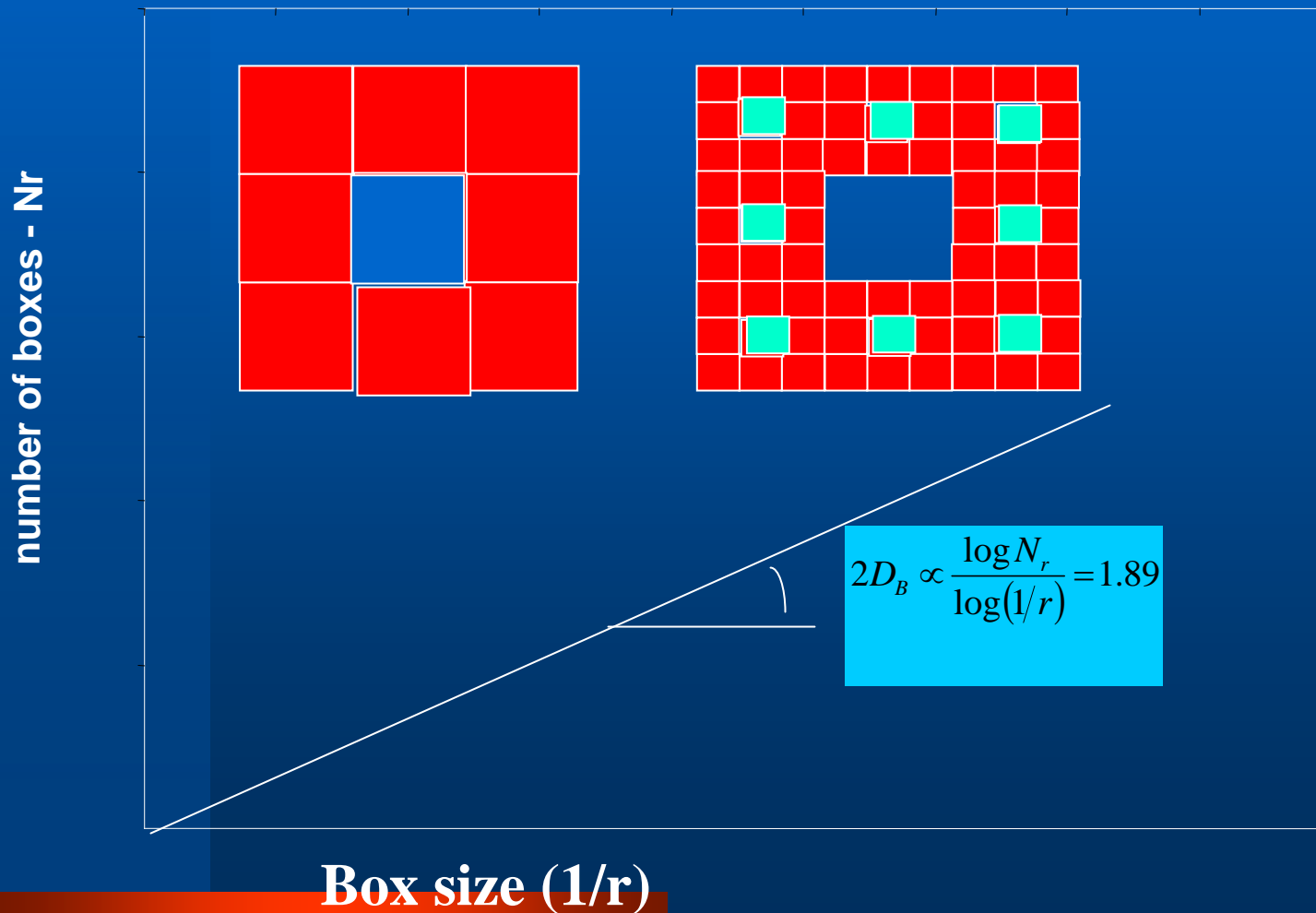


Mathematical Fractals



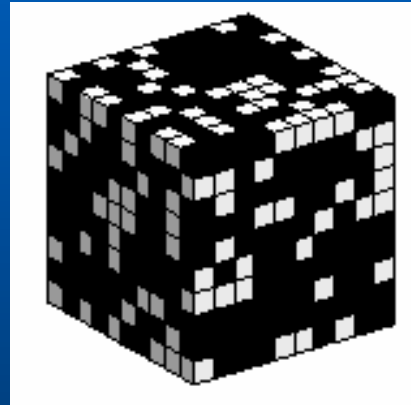
Fractal Dimension in 2D

Box counting method



Fractal Dimension in 3 D

number of 3D boxes - N_r

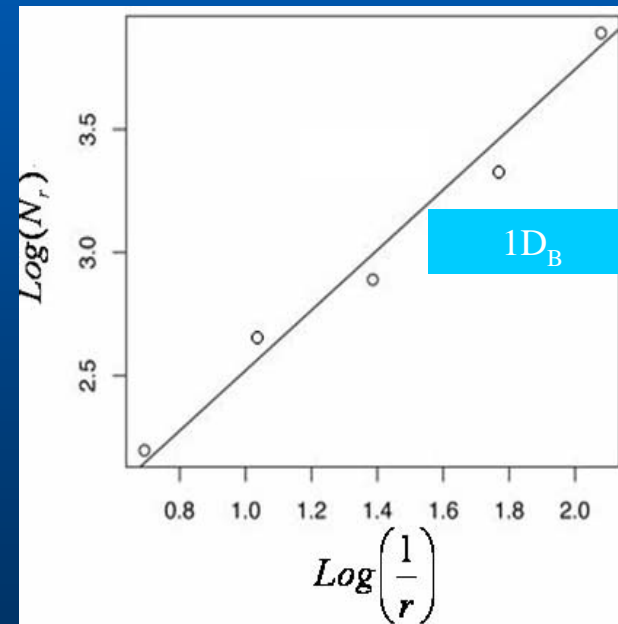
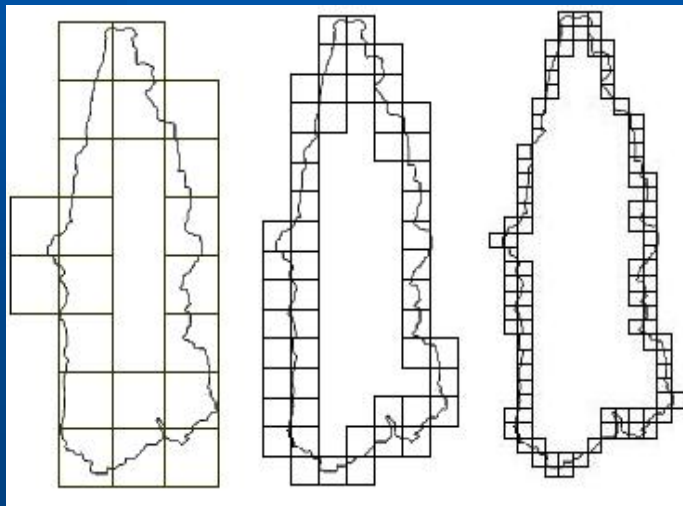


$$3D_B \propto \frac{\log N_r}{\log(1/r)} = 2.7$$

$$D_m = \frac{\log(m_0 N_r)}{\log(1/r)}$$

Box size ($1/r$)

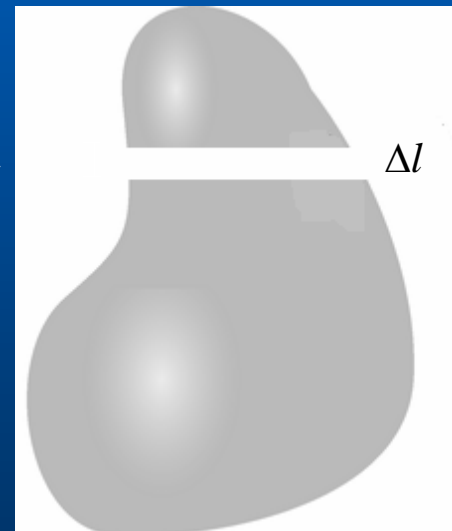
Direct Measurements of Fractal Dimensions on Floc Section Image



Fractal Dimension in 3D

Reconstruction of a Floc in 3D

S – floc cross-sectional area



Indirect Determination of Mass Fractal Dimension (D_m)

$$M = m_o N_r = r^{-D_m}$$

$$D_m \propto \frac{\log(m_o N_r)}{\log(1/r)}$$

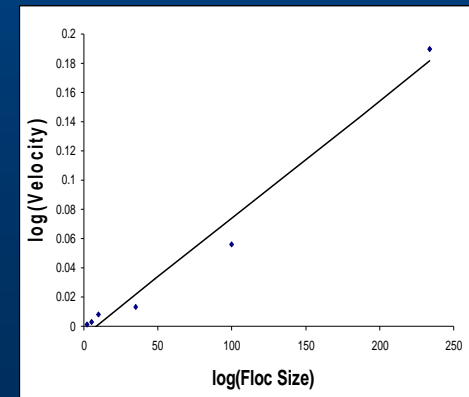
$$M = V\rho \sim r^3 \rho$$

$$\rho \propto (\text{velocity}) / r^2$$

$$-D_m = C + 1$$

Flocs are perfect spheres

Flocs are non-permeable

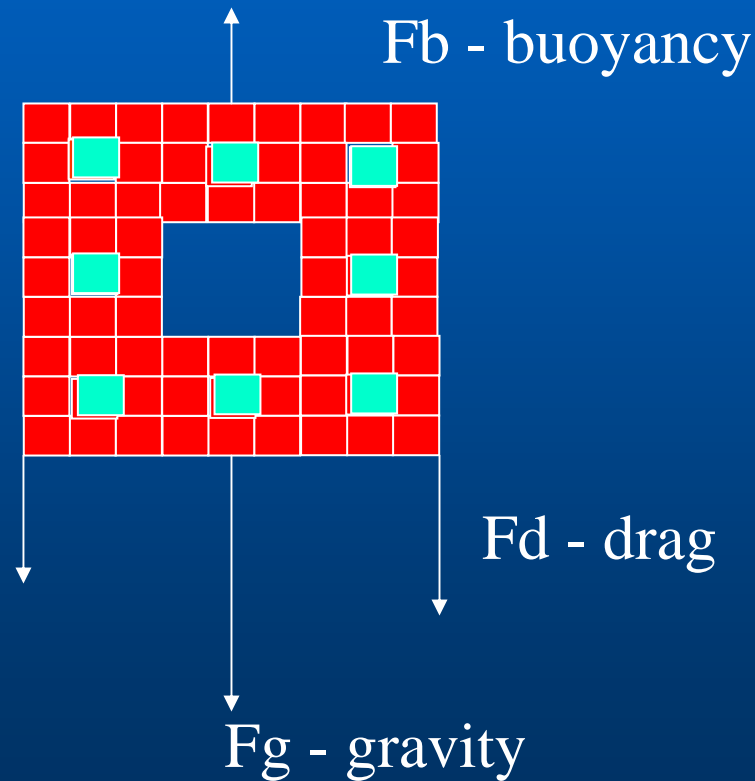


Fractal Dimensions of Chemical Coagulation Floccs

	FRACTAL DIMENSIONS			
	(1D _B)	(2D _B)	(3D _B)	(D _m)
Lime softening floccs (Vahedi & Gorczyca 2008)	1.09	1.91 (1.89)	2.71 (2.70*)	1.82
Alum coagulation floccs (Gorczyca 1991, Gorczyca & Ganczarzyk 1996, 1999)	1.13-1.25	1.91-1.99	N.A	1.37-1.79
Ferric coagulation floccs (Bahrami & Gorczyca 2008)	1.11- 1.16	1.94-1.99	N.A.	N.A.

* Sierpinski Carpet (1/3)

Settling of Chemical Coagulation Flocs

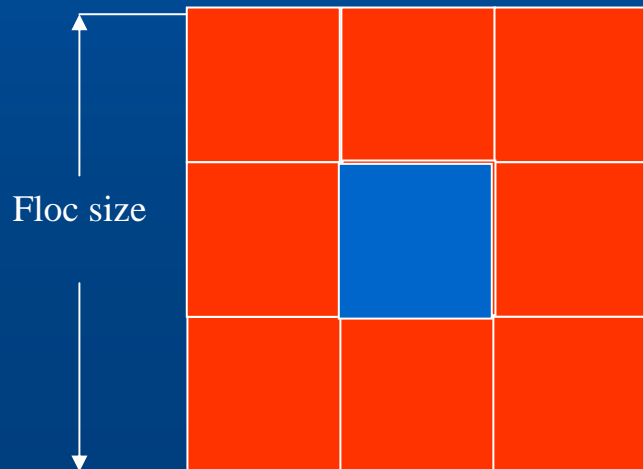


Fractal Permeability Model

(Adler, 1986)

Carman - Kozeny equation for Sierpinski carpet

$$k^* = 1/3.51 \left(\frac{b^2 - l^2 - b}{4l} \right)^2 \left(\frac{b^2 - l^2}{b^4} \right)^N$$



k^* – dimensionless permeability $k^* = k/d_a^2$
 b^2 – number of sub squares in the carpet, $b=3$
 l^2 – number of sub squares removed in the carpet, $l=1$
 N – the construction stage, $N=1$

Summary

Direct microscopic reconstructions of lime softening flocs indicate that Sierpinski Carpet model with constriction ratio of $1/3$ can be used to model structure of these flocs

Modelling of settling of these flocs may be possible with the Stokes' law once it is re-written for this particular Sierpinski carpet

Work on other types of flocs (biological flocs) is needed.

Acknowledgements

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