FRACTAL DIMENSIONS AND SETTLING PROPERTIES OF CHEMICAL COAGULATION FLOCS

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Gravity Settling

Individual flocs

Floc blanket – solid/liquid interface zone settling
Settling of Individual Flocs

Stokes’ Law

\[ V_t^2 = \frac{4g(\rho_F - \rho_w) d}{3 \, \Omega \, C_D \, \rho_w} \]

Density

Size

Drag

Inaccurate Floc Settling Velocity Predictions
Settling of Floc Blanket

\[ V_{zs} = k \exp(-nC) \]

- \( V_{zs} \) – sludge settling velocity
- \( k \) – empirical constant
- \( n \) – empirical constant
- \( C \) – sludge concentration

**K, n, - empirical coefficients specific for each sludge**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Alum Coagulation Sludge</th>
<th>Activated Sludge</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 )</td>
<td>2.57 (Zheng and Bagley, 1999)</td>
<td>13 (Ekama et al., 1997)</td>
</tr>
<tr>
<td>( v_t )</td>
<td>2.52</td>
<td>13.68</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>3.61 ( \times ) 10^5 (Zheng and Bagley, 1999)</td>
<td>3.62 ( \times ) 10^4 (Zheng and Bagley, 1999)</td>
</tr>
<tr>
<td>( (\rho_f - \rho_w) u / k )</td>
<td>2.25 ( \times ) 10^5</td>
<td>9.95 ( \times ) 10^4</td>
</tr>
<tr>
<td>( n_2 )</td>
<td>0.5 (Zheng and Bagley, 1999)</td>
<td>0.64 (Zheng and Bagley, 1999)</td>
</tr>
<tr>
<td>( \rho_s^{-1} )</td>
<td>0.4 (Goreczyca, 1991)</td>
<td>0.71 (Li and Ganczerczyk, 1987)</td>
</tr>
</tbody>
</table>
Objectives of this study

Analyze Properties of Flocs

Model Settling of Flocs
Flocs are Fractals
Mathematical Fractals

1
1
1

1/2
1/2
1/2

1/3
1/3
1/5

1/5
Fractal Dimension in 2D
Box counting method

\[ 2D_f \propto \frac{\log N_r}{\log (1/r)} = 1.89 \]
Fractal Dimension in 3D

$$3D_B \propto \frac{\log N_r}{\log (1/r)} = 2.7$$

$$D_n = \frac{\log (m \times N_r)}{\log (1/r)}$$
Direct Measurements of Fractal Dimensions on Floc Section Image
Fractal Dimension in 3D
Reconstruction of a Floc in 3D

$S$ – floc cross-sectional area
**Indirect Determination of Mass Fractal Dimension \((D_m)\)**

\[
M = m_o N_r = r^{-D_m}
\]

\[
M = V \rho \sim r^3 \rho
\]

\[
\rho \propto \frac{\text{(velocity)}}{r^2}
\]

\[
-D_m = C + 1
\]

- Floccs are perfect spheres
- Floccs are non-permeable
## Fractal Dimensions of Chemical Coagulation Flocs

<table>
<thead>
<tr>
<th></th>
<th>1D&lt;sub&gt;B&lt;/sub&gt;</th>
<th>2D&lt;sub&gt;B&lt;/sub&gt;</th>
<th>3D&lt;sub&gt;B&lt;/sub&gt;</th>
<th>D&lt;sub&gt;m&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lime softening flocs</strong></td>
<td>1.09</td>
<td>1.91</td>
<td>2.71</td>
<td>1.82</td>
</tr>
<tr>
<td>(Vahedi &amp; Gorczyca 2008)</td>
<td></td>
<td>(1.89)</td>
<td>(2.70*)</td>
<td></td>
</tr>
<tr>
<td><strong>Alum coagulation flocs</strong></td>
<td>1.13-1.25</td>
<td>1.91-1.99</td>
<td>N.A</td>
<td>1.37-1.79</td>
</tr>
<tr>
<td>(Gorczyca 1991, Gorczyca &amp; Ganczarczyk 1996, 1999)</td>
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</tr>
<tr>
<td><strong>Ferric coagulation flocs</strong></td>
<td>1.11-1.16</td>
<td>1.94-1.99</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>(Bahrami &amp; Gorczyca 2008)</td>
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</table>

* Sierpinski Carpet (1/3)
Settling of Chemical Coagulation

Flocs

Fg - gravity

Fb - buoyancy

Fd - drag

Fg - gravity
Fractal Permeability Model
(Adler, 1986)

Carman - Kozeny equation for Sierpinski carpet

\[ k^* = \frac{1}{3.51} \left( \frac{(b^2-l^2-b)/4l}{(b^2-l^2)/b^4} \right)^2 \]

- \( k^* \) – dimensionless permeability \( k^* = k/d_a^2 \)
- \( b^2 \) – number of sub squares in the carpet, \( b=3 \)
- \( l^2 \) – number of sub squares removed in the carpet, \( l=1 \)
- \( N \) – the construction stage, \( N=1 \)
Summary

Direct microscopic reconstructions of lime softening flocs indicate that Sierpinski Carpet model with constriction ratio of 1/3 can be used to model structure of these flocs.

Modelling of settling of these flocs may be possible with the Stokes’ law once it is re-written for this particular Sierpinski carpet.

Work on other types of flocs (biological flocs) is needed.
Acknowledgements

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- Shirin Bahrami, Ph.D.