A Conceptual Model for Cryptosporidium Transport in Watersheds

CHAN HEE PARK AND PETER M. HUCK*

Department of Civil Engineering, University of Waterloo, 200 University Avenue West, Waterloo, Ontario N2L 3G1

This paper describes a conceptual model to estimate Cryptosporidium parvum oocyst transport from source to water treatment plant intake. The intent of the model is ultimately to be able to predict oocyst concentrations at an intake to an order-of-magnitude level. The transport and fate mechanisms included are: oocyst detachment from waste or soil, generation of runoff, overland transport, reservoir and in-stream transport, and oocyst die-off. The model is formulated in finite difference form, and deals with both non-point sources from manure-applied areas, and point sources from wastewater treatment plants. An important contribution of this work is the recognition that the settling rates of free and floc- or particle-associated oocysts can be considerably different. This has important implications for their transport.

A finite difference scheme was developed for five sections of a hypothetical watershed: a point source, a lake or reservoir (which can be modelled as either a continuous stirred tank reactor or an ideal rectangular setting tank), the section of stream channel from the outlet of the lake or reservoir to the confluence with another stream, a tributary with a non-point source, and the stream section from the confluence to a water treatment plant intake. The stream confluence is handled with a simple mass and flow balance. It would be very expensive to collect the necessary data to test the model. Because an appropriate data set was not available, the model was tested by means of a sensitivity analysis for the hypothetical watershed, using reasonable parameter settings for the base case.

The major contribution of the model is in defining the mechanisms involved in oocyst transport within a watershed. It gives important insights into the significance of various factors, provides a basis for data collection, and identifies areas where experimental investigations are required to avoid the need for simplifying assumptions. At its current state of development, the model cannot be used to provide quantitative predictions, but defines a base from which further detailed modelling can be developed to aid in decision-making for pathogen control. Using the framework that this model provides, contributions from other sources of Cryptosporidium oocysts such as domestic animals and combined sewage overflows could also be modelled.

Key words: Cryptosporidium, oocysts, oocyst transport, modelling, drinking water, point source, non-point source, sensitivity analysis

* Corresponding author; pm2huck@uwaterloo.ca
Introduction

Cryptosporidium parvum has been implicated in a number of waterborne outbreaks of gastroenteritis. The most notorious outbreak was in Milwaukee, in 1993. More than 400,000 people were affected (MacKenzie et al. 1994), and it was the largest outbreak of waterborne disease ever recorded in the U.S.A. In some outbreaks, inadequacies in the water treatment process have been blamed for passage of oocysts into the water supply.

Cryptosporidium parvum exists in the environment in the form of an oocyst. These oocysts are commonly detected in surface waters, whether the waters are pristine/near pristine or affected by human activity. Some surveys have found more than 70% of the waters sampled to be oocyst-positive (LeChevallier et al. 1991; Rose et al. 1991; Lisle and Rose 1995). Major oocyst sources include animal husbandry operations and sewage treatment effluents. Studies have shown that the concentration of Cryptosporidium oocysts is related to hydrologic events (Stewart et al. 1997; Atherholt et al. 1998) such as rainfall and runoff, and to watershed characteristics (Hansen and Ongerth 1991; Kelley et al. 1995; Ong et al. 1996). Therefore, an understanding of source terms and factors affecting oocyst transport is required to estimate raw water oocyst concentrations at drinking water intakes. In terms of transport, hydrologic events and watershed and watercourse characteristics are important.

Because of the difficulty in inactivating Cryptosporidium with conventional disinfectants, enhanced treatment is often required to eliminate this pathogen effectively. The extent of treatment required is determined by the levels of pathogens in the raw water. In this regard, it is not only the average concentration that is important, but also the expected peak values. Knowing the significance of parameters affecting Cryptosporidium oocyst transport and developing models to estimate the concentrations at a drinking water intake dynamically are important in providing public health protection.

To date, little work has been done to describe oocyst transport both to and within watercourses, although various models describing sediment transport, hydrologic processes and contaminant transport (chemical or other types of microorganisms) exist.

This paper describes the development of a conceptual model for Cryptosporidium oocyst transport in a watershed. The long-term goal is to apply the model to specific watersheds in order to predict approximate peak oocyst concentration ranges to be expected at given water treatment plant intakes. This would require further model refinement and experimental work to quantify certain assumed model parameters and provide calibration data. Such a model likely could only predict oocyst concentrations to an order-of-magnitude level. However, predictions at this level are what is required for the design of water treatment facilities, because treatment requirements are related to order-of-magnitude oocyst concentrations in the raw water.
The specific objectives of this paper are: i) to describe the role of hydrologic parameters such as rainfall, runoff, and lake residence time with respect to oocyst transport in a watershed; ii) to establish an oocyst-specific conceptual model that could also be used to describe transport of other pathogens; iii) to present the finite difference scheme for the model; and iv) to test the model using a sensitivity analysis.

A sensitivity analysis using a hypothetical watershed was the approach used for testing the model because an appropriate data set was not available. Such a data set would cost on the order of several hundred thousand dollars to obtain. Although the focus of modelling is Cryptosporidium parvum, the more general term Cryptosporidium is used in this paper.

Background

Physical Behaviour of Cryptosporidium Oocysts

Cryptosporidium oocysts may be considered as organic particles which have their own physical properties. Oocysts have a spherical shape, a diameter of 4 to 6 µm and a density of 1.05 g/cm³ (Medema et al. 1998). According to Drozd and Schwartzbrod (1996), oocysts do not demonstrate marked hydrophobic properties. These authors observed a zeta potential close to −25mV at pH 6 to 6.5 in deionized water, which is a little higher than −10 mV measured in natural water (Gregory 1994). Animal waste or oocysts on manure-applied areas can be thought of as a form of physical mixture of oocysts and waste or soil. Because of the surface properties of oocysts, their detachment from waste to water may be likely during runoff.

Oocysts can exist in water in two states, a) detached, or b) attached to other suspended particles. It is important to consider their transport in a watershed separately, as this affects processes such as coagulation, flocculation, and sedimentation.

Medema et al. (1998) compared experimentally determined sedimentation velocities of free and attached oocysts with calculated velocities (Stokes’ law) based on oocyst size and density, and the density and viscosity of the sedimentation medium. The theoretically calculated sedimentation rates showed good agreement with the experimentally observed rates. The initial apparent sedimentation velocity of free oocysts in Hanks balanced salt solution at 23°C was 0.35 µm/s (1.26 mm/h), which is extremely low. Considering a hypothetical example of runoff lasting one hour and having a depth of about 1 cm, it is reasonable to expect significant sedimentation of free oocysts would likely occur in lakes and reservoirs. Consequently, free oocysts could be transported quite a long way. Medema et al. (1998) also showed that oocysts in the surface water environment may be attached to other particles, which will affect their sedimentation rate. These authors mixed oocysts with settled secondary effluent (biologically treated sewage) and found that oocysts readily attached to the (biological) particles in the effluent. Thirty percent of the oocysts
attached during the first minutes of mixing, and this fraction increased to approximately 75% after 24 h. The sedimentation velocity of oocysts attached to secondary effluent particles increased with particle size and was determined by the sedimentation rate of the effluent particles.

**Existing Non-point Source Contaminant Models**

The Agricultural Nonpoint Source Pollution Model (AGNPS) was developed to compare the effects of different watershed pollution control management practices (Young et al. 1986). AGNPS simulates sediment and nutrient loadings from agricultural watersheds for single storm events or for continuous data input. Watersheds in the model are discretized into a series of square cells, for which homogeneous characteristic parameters are assigned. AGNPS is partitioned into two submodels. The erosion portion of the model provides estimates of upland erosion, channel erosion, and sediment yield. The model uses the Modified Universal Soil Loss Equation (Williams 1975) for soil erosion calculations and distributes predicted erosion into five particle size categories: sand, silt, clay, small aggregates, and large aggregates. The pollutant transport portion of AGNPS addresses only soluble pollutants, and loads are determined using relationships between chemical concentrations, sediment yield, and runoff volume (Young et al. 1986). The hydrology is calculated by the Soil Conservation Service (SCS) runoff curve number method (SCS 1972).

The Chemicals, Runoff, and Erosion from Agricultural Management Systems (CREAMS) model was developed to aid in the assessment of agricultural best management practices for pollution control (Knisel 1980). CREAMS is a continuous simulation model, requiring continuous precipitation data and monthly values of air temperature and solar radiation. Soil and crop type data are also provided as inputs. In order to assess best management practices, the user can simulate various management activities, such as aerial spraying or ground application of pesticides, animal waste management, tillage operations, or terracing (Knisel 1980).

CREAMS calculates runoff volume, peak flow, infiltration, evapotranspiration, soil water content, and percolation on a daily basis. Daily erosion and sediment yield are also estimated and average concentrations of sediment-associated and solute chemicals are calculated for the runoff, sediment, and percolating water (Knisel 1980). CREAMS also uses the SCS curve number method. The erosion component of the model considers the basic processes of soil detachment (Universal Soil Loss Equation, USLE [Wischmeier and Smith 1978]), transport, and deposition.

The ANSWERS (Areal Nonpoint Source Watershed Environment Response Simulation) is a distributed parameter, event-based model for predicting the hydrologic and erosion response of agricultural watersheds (Beasley and Huggins 1981). The watershed is divided into uniform square elements. Within each element the model simulates processes of interception, infiltration, surface storage, surface flow, sediment detachment (USLE) and transport. It is primarily a runoff and sediment
model; the nutrient simulation is based on a simple correlation between chemical concentrations, sediment yield, and runoff volume.

For four separate rainfall events, the simulated (ANSWERS) hydrologic responses were found to correlate closely with actual hydrograph responses in the watershed. Predicted sediment loads from ANSWERS, however, were significantly and consistently less than actual measured loads (Engel et al. 1993). Those authors showed that rough estimates for ANSWERS input parameters were sufficient for the prediction of hydrologic response, but not for predicting sediment loads.

The Soil Water and Assessment Tool (SWAT; Arnold et al. 1993) was developed as an extension to the Simulator for Water Resources in Rural Basins (SWRRB; Williams et al. 1985). SWAT is a continuous spatially distributed watershed model operating on a daily time step. It simulates runoff, sediment, nutrient, and pesticide movement through a watershed and aids in assessing water supplies and non-point source pollution in large basins (Arnold et al. 1993). SWAT was one of the non-point source pollution water quality models assessed in a comparison conducted by Engel et al. (1993). SWAT estimates for total runoff and nutrient and sediment loads were less accurate than the ANSWERS simulated values.

The Enhanced Stream Water Quality Model (QUAL2E), although not a non-point source model, is used for modelling stream water quality. It is applicable to well-mixed dendritic streams and simulates the major reactions of nutrient cycles, algal production, benthic and carbonaceous demand, atmospheric re-aeration and their effects on the dissolved oxygen balance. It can predict up to 15 water quality constituent concentrations. It is intended as a water quality planning tool for developing total maximum daily loads and can also be used in conjunction with field sampling for identifying the magnitude and quality characteristics of non-point sources. By operating the model dynamically, the user can study diurnal dissolved oxygen variations and algal growth, but the effects of dynamic forcing functions, such as headwater flows or point source loads, cannot be modelled. QUAL2EU is an enhancement allowing users to perform three types of uncertainty analyses: sensitivity analysis, first order error analysis, and Monte Carlo simulation (Brown and Barnwell 1987).

QUAL2E, which can be operated either as a steady-state or as a dynamic model, assumes that the major transport mechanisms, advection and dispersion, are significant only along the main direction of flow (longitudinal axis of the stream or canal). It allows for multiple waste discharges, withdrawals, tributary flows, and incremental inflow and outflow. It also has the capability to compute required dilution flows for flow augmentation to meet any pre-specified dissolved oxygen level.

Limitations of Existing Models

Saunders and Maidment (1996) performed a GIS assessment of non-point source pollution in the San Antonio–Nueces coastal basin in the U.S.A. They evaluated four constituents: phosphorus, nitrogen, cadmium,
and fecal coliforms. Predicted concentrations for phosphorus and nitrogen from the simulated point sources matched closely with average observed concentrations in the basin. Predicted fecal coliform concentrations, however, did not match well with average observed values. This may be because this model was not intended for fecal coliforms.

Watershed level modelling is complex and involves various parameters. Most of the existing models predict common processes such as runoff estimation, soil detachment and water channel transport. Although these processes also affect Cryptosporidium oocyst transport in a watershed, existing models were developed to deal with sediments, nutrients and pesticides.

There has been no model developed specifically for pathogens such as Cryptosporidium oocysts in a watershed. Existing models deal with sediments as a whole, which is insufficient for modelling oocyst transport. With these models, it is not possible to distinguish oocysts from other transported sediment particles of the same size. Although some existing models do incorporate bacteria, these models do not include aspects that are important regarding Cryptosporidium, such as the distinction between free and floc-associated oocysts. It is therefore important to consider oocyst transport at the conceptual level, because there are some fundamental differences from the transport processes for other contaminants in a watershed.

**Approach**

Because of the difficulties identified in applying existing models to describe Cryptosporidium oocyst transport in a watershed, a conceptual modelling approach based on first principles was adopted. The conceptual model assumes oocysts to behave as particles, but which exhibit die-off. Although to some extent the relationships are similar to those in sediment transport models such as AGNPS and ANSWERS, as noted previously those models deal with sediments as a whole, and do not allow oocysts to be distinguished from other transported sediment particles of the same size. The following subsections describe the various mechanisms included in the conceptual model.

**Transfer from Solids to Water**

In overland flow, oocysts can be transported either as free-floating, or as floc-incorporated or particle-associated entities. The ratio of free-floating to floc-incorporated oocysts in runoff is a function of several factors. Manure-incorporated oocysts are not likely to be chemically bonded to the other fecal material, but rather are physically attached or trapped. Being of a size between that of clay and silt and having nearly neutral buoyancy (using textural classifications presented by Brady [1991]), oocysts would be entrained easily and suspended in runoff water. However, they may have a high affinity for mineral and organic fractions
of the soil, effectively changing their diameter and mass for transport purposes. The threshold of kinetic energy needed to entrain and transport a particle in flowing water is a function of the particle’s effective diameter and mass.

Oocysts may be present in waste deposited on the surface directly from animal droppings, or from manure applied on the top of the soil. Oocysts may also be in the soil, because some manure is incorporated into the soil when it is applied. In this case, oocysts could penetrate deeper into the soil due to rainfall and become less likely to be transported into surface runoff. Because of an oocyst’s relatively large size compared to bacteria, it is much more difficult for oocysts to move through soil or into groundwater than bacteria unless there are fractures in the subsurface or extensive tile drainage. Thus, Couillard and Li (1993) showed that applying manure in the soil can improve water quality, compared to surface application. For modelling purposes, it is considered reasonable to neglect transport through the subsurface (unless there is fracturing or extensive tile drainage), for oocysts applied in the soil.

Quantification of Detachment

Runoff begins to generate shear forces upon oocysts in the soil when rainfall rate exceeds the infiltration rate. This shear force can differ from that produced by the slope of the terrain.

Bacterial detachment is commonly modelled as a kinetic process, and solely related to the sorbed concentration of bacteria (Bales et al. 1991; Harvey and Garabedian 1991; Lindqvist et al. 1994). Experimental observations, however, have demonstrated that detachment also is a function of bacterial residence time and is not solely dependent upon attached concentration (Escher 1986, as cited by Escher and Characklis 1990). For the model developed in this paper, oocyst detachment is assumed to be analogous to bacterial detachment.

Bacterial detachment as a function of time has been described by a single-term exponential decay model (Escher 1986). Using a similar approach, oocyst detachment can be described as follows:

\[
\frac{N}{N_0} = e^{-wt_i} \quad t < t_i
\]

\[
N = \text{const} \quad t \geq t_i
\]

where \(N\) is the number of oocysts remaining in the waste at runoff time \(t\) (T), \(N_0\) is the initial number of oocysts in the waste at the time of initial runoff, and \(w\) is an exponential desorption rate constant (T\(^{-1}\)).

\(N\) is set to a constant after a specified time, \(t_i\) (T), in order to provide for a fraction of oocysts that may be considered irreversibly attached after that time. Higher values of \(w\) increase the rate of detachment.
A two-rate desorption model describes the phenomenon that a large number of oocysts are quickly released initially, followed by a slower release. The model is given as:

\[
\frac{N}{N_0} = A e^{-k_1 t} + (1 - A) e^{-k_2 t}
\]  

(2)

where \(k_1\) and \(k_2\) are the fast and slow desorption rate coefficients (T\(^{-1}\)), respectively, and \(A\) is a weighting factor.

For high values of \(A\), the shift in domination from fast desorption to slow desorption occurs at a high value of \(N/N_0\). For lower values of \(A\), this shift occurs at a lower value of \(N/N_0\).

**Settling Rate**

As noted previously, the effective settling rate of an oocyst is influenced strongly by whether it is present in a free state or floc-associated. According to Droppo and Ongley (1993), the average floc size of the Grand River in Ontario is 9.1 µm. Even though the characteristics of overland flow are different from those of the Grand River, in the absence of better information, this diameter will be used here for floc-associated oocysts. For purposes of the present work the exact floc size does not have a large impact on the results; the fraction of free versus floc-associated oocysts is far more important.

The settling of dilute suspensions in water follows Stokes' law, provided the flow is laminar and minimal interaction between particles is assumed. Stokes’ equation is given by:

\[
V_s = \frac{g}{18\mu} (r_s - r_d) d^2
\]

(3)

where \(V_s\) is settling velocity (LT\(^{-1}\)), \(g\) is acceleration due to gravity (LT\(^{-2}\)), \(\mu\) is dynamic viscosity (ML\(^{-1}\) T\(^{-1}\)), \(\rho_s\) is mass density of the particle (ML\(^{-3}\)), \(\rho\) is mass density of water (ML\(^{-3}\)), \(d\) is diameter of the particle (L), and where \(L\) is length and \(M\) is mass.

Assuming that most overland flow is laminar (Harry 1970), Stokes’ law can be used to estimate oocyst settling rates in this flow. The calculated oocyst settling velocity therefore would be 1.5 to 3.5 mm/h for a size range 4 to 6 µm and density of 1.05 g/cm\(^3\) (size and density values are taken from Medema et al. [1998]). In contrast, for a size of 9.1 µm (the Grand River average floc size) and assumed density of 1.5 g/cm\(^3\), the floc settling rate would be 77 mm/h (the density 1.5 is based on a sediment density of 2.6 g/cm\(^3\) [clay] and an oocyst density of 1.05 g/cm\(^3\). A more accurate floc density would not affect the point under discussion). The settling velocity of floc-associated oocysts thus estimated by Stokes’ law is more than an order of magnitude higher than that of free oocysts. This
is due to both the larger diameter and higher density of the floc-associated oocysts. Therefore, based on theoretical calculations, more floc-incorporated oocysts would settle out than free-floating oocysts, because of the generally shallow depth of runoff.

Although scouring of settled oocysts will occur, it is possible that several storms may be required for floc-incorporated oocysts to reach a watercourse. Depending on the die-off rate of oocysts, only a small fraction of floc-associated oocysts may be viable if they reach a water treatment plant as a result of a number of storm events. This would be particularly true if these storm events are infrequent. However, floc-associated oocysts near a water body should be considered in calculations, especially when a high number of such oocysts may be present.

**Overland Transport**

If the rate of oocyst transfer from solids to runoff water is obtained or calculated by the equations presented earlier, the flow velocity is determined by Manning’s equation, and the number of free-floating and floc-associated oocysts present on areas near a water body are estimated, the oocyst transportation rate can be written as:

\[
N_{\text{free}} = (C_{\text{free}})Q
\]

\[
N_{\text{floc}} = (C_{\text{floc}})Q
\]

\[
N_t = (C_{\text{free}})Q + (C_{\text{floc}})Q
\]

where \(N_{\text{free}}\) is number of free-floating oocysts transferred per unit time \(T^{-1}\), \(C_{\text{free}}\) is concentration of free-floating oocysts (number of free-floating oocysts/volume of runoff water) \(L^{-3}\), \(N_{\text{floc}}\) is number of floc-associated oocysts transferred per unit time \(T^{-1}\), \(C_{\text{floc}}\) is concentration of floc-associated oocysts (number of particulate-associated oocysts/volume of runoff water) \(L^{-3}\), \(Q\) is overland flow rate (volume of runoff water per unit time) \(L^3T^{-1}\), and \(N_t\) is total number of oocysts transferred per unit time \(T^{-1}\).

**Water Channel Transport**

Water channel transport is based on the advection-dispersion equation. Therefore, the governing equation describing water channel transport of oocysts (including a die-off function) is:

\[
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - v \frac{\partial C}{\partial x} - kC + S(x, t)
\]
where $C$ is the concentration of oocysts (number of oocysts/unit volume) ($L^{-3}$), $D$ is the dispersion coefficient for the oocysts ($L^2T^{-1}$), $v$ is the flow velocity ($LT^{-1}$), $k$ is the die-off coefficient ($T^{-1}$), and $S$ is the source function ($L^{-3}T^{-1}$), which can be a point or non-point source in each stream.

Walker et al. (1998) reported that a contamination event represents a short-duration pulse of oocysts in raw water supply systems that is likely to be related directly to a hydrologic event. Since oocyst transport in a watershed is driven largely by hydrologic events, the transport must be represented by a dynamic (transient) state model. Therefore the left-hand side of the equation is not zero.

### Die-off (Viability)

Many pathogenic microorganisms that occur in animal wastes survive well in soils but are quickly inactivated in natural waters (Maas et al. 1987). Sattar et al. (1999) investigated various factors affecting inactivation. Investigations of the survival of *C. parvum* in soil and water show the influence of the type and intensity of stresses encountered in each environment. Under some conditions, loss of viability can be very slow. Robertson et al. (1992) did experiments involving oocyst immersion in river water, which indicated that half the initial number of oocysts would remain infective after ~30 days (estimated by assuming first-order decay).

Walker et al. (1998) summarized oocyst die-off for incubation experiments in feces. Jenkins et al. (1997) also did experiments and gathered information about inactivation of *C. parvum* oocysts stored in fecal pools or water at various temperatures. It is very difficult to apply a die-off function to each stage that oocysts go through as a result of hydrologic events. Aside from site-specific sampling and analysis, there is no way to know the viability of oocysts on a given field. However, information on oocyst viability in both feces and natural water can be used to model oocyst die-off, and assumptions can be made regarding the viability of oocysts found on a field.

By assuming that oocyst die-off is a first-order process, oocyst transport in a water channel can be modelled by adding a first-order decay term to the advection-dispersion equation. Because the decay rate constant would be dependent on factors such as temperature, at a later stage of model development a formal means of including such factors should be incorporated.

### Oocyst Behaviour in Lakes

Lakes and reservoirs are natural coagulation and sedimentation basins. With hydraulic residence times often ranging from months to years, considerable sedimentation can occur. Natural aggregation increases particles sizes and thus settling velocities, accelerates particle removal to bottom sediments, and decreases the concentrations of particles in the water column (Weilenmann et al. 1989). Two important fac-
tors affecting sedimentation in a lake or reservoir are: (i) the hydraulic loading or overflow rate and (ii) the colloidal stability or sticking factor of the particles.

Two simple models that can be used to approximate the behaviour of oocysts in lakes and reservoirs are the continuous stirred tank reactor (CSTR) model and the ideal rectangular settling tank (IRST) model.

**Continuous stirred tank reactor**

A CSTR model by nature implies mixing. It is useful in approximating the bulk water behaviour of small lakes and reservoirs, especially where the inflow and outflow are relatively large in relation to the volume. Oocysts in a lake or reservoir will settle relative to the bulk water at a velocity that can be calculated by Stokes’ law. Even though there is mixing in a lake, it is a relatively stationary water body in comparison to a river. It is reasonable to assume that there is a laminar flow, unless wind or other disturbances are present. Therefore, the settling of oocysts as a primary loss mechanism in a lake should be incorporated into the CSTR model. The laminar flow assumption may imply a slight departure from the perfect mixing assumed for an ideal CSTR.

The governing mass balance equation for a lake modelled as a CSTR, including a settling term, is given by:

\[
\frac{dC_{\text{lake}}}{dt} = Q_{\text{in}}C_{\text{in}} - Q_{\text{out}}C_{\text{lake}} - \frac{v_sA_sC_{\text{lake}}}{\text{lake}} - k'' C_{\text{lake}} \tag{8}
\]

where \(\text{lake}\) is the volume of lake (L³), \(Q_{\text{in}}\) is the inflow (L³T⁻¹), \(Q_{\text{out}}\) is the outflow (L³T⁻¹) (\(Q_{\text{in}}\) and \(Q_{\text{out}}\) are assumed to be equal), \(v_s\) is the settling velocity of the oocysts (LT⁻¹), \(A_s\) is the area of the lake (L²), \(k\) is the first-order oocyst die-off coefficient (T⁻¹), \(C_{\text{in}}\) is the inflow oocyst concentration (L⁻³), and \(C_{\text{lake}}\) is the oocyst concentration (L⁻³), both in the outflow and in the lake itself.

Assuming complete mixing, equation 8 can be rewritten as:

\[
\frac{dC}{dt} + 1C = \frac{Q}{n}C_{\text{in}} \tag{9}
\]

where \(C\) is the concentration of oocysts in the lake (and in the outflow), \(Q\) is the input and output flow, \(C_{\text{in}}\) is the input concentration of oocysts, and \(\lambda\) can be derived (Chapra 1997) from equation 8.

\[
1 = \frac{Q + v_sA_s + k''}{n} \tag{10}
\]

Equation 9 shows that the output oocyst concentration from a lake is a function of the input concentration. The complete mixing assumption of the CSTR model implies that, regardless of lake size or retention time, oocysts will start to appear in the outflow as soon as they enter the lake.
Since it is a transient state model, the CSTR model is preferable to the IRST model discussed below for modelling dynamically changing oocyst inputs into a lake. The oocyst residence time \( t \) is affected by die-off and settling in addition to the rate of flow. Chapra (1997) defined a contaminant residence time as:

\[
t = \frac{1}{1} \tag{11}
\]

**Ideal rectangular settling tank**

The theory of an ideal rectangular settling tank (e.g., Reynolds 1982) is well known with respect to discrete settling. In theory, one can select a terminal (settling) velocity, \( V_s \) (L T\(^{-1}\)), and design the settling basin such that all particles with a terminal velocity greater than \( V_s \) will be removed.

The overflow rate is given by:

\[
V_s = \frac{Q}{A_s} \tag{12}
\]

where \( Q \) represents both inflow and outflow (L\(^3\)T\(^{-1}\)), and \( A_s \) is the surface area of the basin (L\(^2\)).

The overflow rate, \( V_s \), is equivalent to the settling velocity of the smallest particle that is 100% removed. If the settling velocity, \( V_{\text{oocyst}} \), is less than \( V_s \), the fraction removed, \( R_{\text{oocyst}} \), is equal to \( V_{\text{oocyst}} / V_s \). Thus,

\[
R_{\text{oocyst}} = \frac{V_{\text{oocyst}}}{V_s} \tag{13}
\]

As an initial approximation, oocyst sedimentation in a lake or reservoir can be estimated using an IRST approach, by using the settling velocities of free and floc-associated oocysts, and the dimensions and inflow/outflow (Q) of the lake. An IRST lake model would also contain an oocyst die-off term.

**Comparison of CSTR and IRST approaches**

For the same constant input concentration, the outlet concentration of a CSTR at steady state would be the same as for an IRST, neglecting minor differences in die-off between the two approaches. Due to the mixing assumptions in the CSTR model, it is difficult to use it to represent a lake with a long retention time. However, for a small lake or reservoir, it is likely that the CSTR model would better explain the dynamics than would the IRST model. The IRST model would give a higher outlet concentration following a residence time corresponding to that for an input to move through the lake in plug flow. Therefore, it is likely appropriate to use the IRST approach to provide a worst case estimate.
The Conceptual Cryptosporidium Transport Model

Overview

The watershed-level conceptual Cryptosporidium transport model (Fig. 1) includes the following mechanisms: oocyst detachment, runoff, overland transport, lake transport, water channel transport and stream confluence (the meeting of two streams). By estimating the impact on oocyst concentration of each mechanism or step, the model helps to determine their relative importance. Following a description of the sample watershed, this section addresses the sequential transport of oocysts initiated by a storm event: runoff assessment, overland velocity calculation, point and non-point source calculation, discharge time control of a non-point source in overland transport, and finally travel distance control of a non-point source in overland transport. The equations used in the model are given in Table 1. The assumptions that are made because no data are available are discussed and are summarized in Table 2. In the future, further development of the model would allow elimination of some of the simplifying assumptions that are currently required.

Sample or Hypothetical Watershed

In order to proceed with the model, a sample or hypothetical watershed was necessary since there were no suitable data available from a real watershed for use in this research. The hypothetical watershed includes both a point source and a non-point source of oocysts, a lake and a stream confluence (Fig. 2). The point source is the discharge from a wastewater treatment plant and the non-point source is an area to which manure has been applied. For the non-point source a rainfall event and the ensuing runoff are the causes of oocyst delivery to a water channel. Then, the water flows through the channel to the intake of the water treatment plant. A lake is located in one of the water channels to illustrate its effect on oocyst concentrations. There is also a stream confluence. Detailed
Table 1. Equations used in the conceptual model

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Equation</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
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<td>Settling rate</td>
<td>( V_s = \frac{q}{18m} (x_s - x) d^2 )</td>
<td>Stoke’s law</td>
</tr>
<tr>
<td>Runoff calculation</td>
<td>SCS Curve Procedure</td>
<td>SCS</td>
</tr>
<tr>
<td>Overland transport</td>
<td>( v = \frac{1}{n} R^2 S / L )</td>
<td>Manning’s equation</td>
</tr>
<tr>
<td></td>
<td>( \frac{N}{N_0} = e^{-\nu t}, t &lt; t_i )</td>
<td>Single exponential function</td>
</tr>
<tr>
<td></td>
<td>( N = \text{const}, t \geq t_i )</td>
<td></td>
</tr>
<tr>
<td>Detachment</td>
<td>( \frac{N}{N_0} = A e^{-k t} + (1 - A) e^{-k t} )</td>
<td>Two-term exponential model</td>
</tr>
<tr>
<td></td>
<td>10% detachment</td>
<td>Simple assumption</td>
</tr>
<tr>
<td>Source calculation</td>
<td>( S (x, t) = \frac{W}{\lambda_{\text{source}}} \frac{d}{dt} \frac{(x - a) H(t) - H(t - t_{\text{transport}})}{t} )</td>
<td>Point source</td>
</tr>
<tr>
<td></td>
<td>( S (x, t) = \frac{W}{\lambda_{\text{source}}} { (x - a) \cdot H(t - t_{\text{transport}}) + H(t - a) } )</td>
<td>Non-point source</td>
</tr>
<tr>
<td>Oocyst flux</td>
<td>( W(t) = Q(t)C(t) )</td>
<td>Mass balance</td>
</tr>
<tr>
<td>Water channel transport</td>
<td>( \frac{\partial N}{\partial t} = D \frac{\partial^2 N}{\partial x^2} - \nu \frac{\partial N}{\partial x} - kN + S(x, t) )</td>
<td>The advection dispersion equation</td>
</tr>
<tr>
<td>Stream confluence</td>
<td>( N_3Q_3 + N_4Q_4 = N_5Q_5 )</td>
<td>Adventive mass balance</td>
</tr>
<tr>
<td></td>
<td>( Q_3 + Q_4 = Q_5 )</td>
<td></td>
</tr>
<tr>
<td>Lake transport</td>
<td>( \frac{\partial c_{\text{lake}}}{\partial t} = -q_{\text{in}} c_{\text{lake}} + q_{\text{out}} c_{\text{lake}} - \nu A_{\text{lake}} c_{\text{lake}} - k c_{\text{lake}} )</td>
<td>CSTR</td>
</tr>
<tr>
<td></td>
<td>( R_{\text{removal}} = \frac{V_s}{V_o} )</td>
<td>IRST</td>
</tr>
</tbody>
</table>
modelling of the confluence itself is very complicated. However, since the model is designed to give the relative importance of the various parameters involved, a simple mass balance approach is used. This adequately represents concentrations downstream of the confluence once complete mixing of the two streams has occurred.

The hypothetical watershed was created using a simplified part of the Grand River watershed, in Ontario, Canada. Therefore the scale, as well as the geometry of the lake and stream are similar to those in the
Runoff Assessment

The conceptual model employs the most commonly used method to estimate runoff, the U.S. Soil Conservation Service (SCS) curve number procedure (SCS 1972). Basically, the SCS method takes into account the infiltration capacity of a given type of soil, land use, and the soil water conditions at the start of a rainfall event (antecedent soil water condition).

Overland Velocity Calculation

When rainfall generates runoff, overland flow velocity needs to be estimated to understand how and when oocysts on a field reach a water channel. The conceptual model uses Manning’s equation, which describes the effect of flow resistance caused by channel roughness:

\[ V = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \]  

(14)

where \( V \) is the average flow velocity in metres per second, \( n \) is Manning’s roughness coefficient, \( R \) is the hydraulic radius in metres, and \( S \) is the dimensionless energy slope.

The hydraulic radius is defined as the flow cross-sectional area divided by the wetted perimeter. For a wide channel (width >20 times the depth), which would apply for overland flow, the hydraulic radius is approximately the same as the flow depth.
Source Calculation

As mentioned previously, there are two types of source functions for oocysts in the sample watershed: point and non-point. A wastewater treatment plant discharge is used here as an example of a point source; overland runoff and areas to which manure has been applied as non-point sources. In the conceptual model, a point source is represented by a dirac delta function while a non-point source is represented by a line load function.

Discharge Time Control for a Non-point Source in Overland Transport

Overland flow velocity is one of the important factors used to estimate non-point source discharge time. Since overland transport is assumed to be caused mainly by the advective term, the velocity at which oocysts are transported to the water channel can be estimated by the overland flow velocity. An important variable is the duration of runoff, which is not simple to calculate, since it is a function of land use, soil type, storm event, etc. For the sample watershed case, the duration of runoff is assumed to be one hour. (A model sensitivity analysis is provided later in this paper.) The distance of a manure-applied area from the water channel affects the discharge time of that non-point source. For example, if it takes 50 min for oocysts to reach the water channel, then the oocyst discharge time is 10 min; i.e., the total runoff time (60 min) minus the oocyst travel time (50 min).

Currently, the conceptual model starts with the oocyst concentration existing on the top of the manure-applied area at the beginning of runoff. At this stage of development the model does not include a function to describe oocysts reaching the manure-applied area as a result of their detachment into the runoff water somewhere up-gradient.

The time during which runoff water passes over the manure-applied area can be shorter than the discharge time, if the length of the manure-applied area in the direction of flow is small. Suppose, continuing the example above, that it only takes the runoff three minutes to pass through the manure-applied area, then, there is only a three-minute discharge of oocysts, even though the difference between the runoff time and the travel time would allow 10 min. Fig. 3 illustrates the two different cases for what determines the non-point source discharge time. As a check, the conceptual model calculates the actual non-point source discharge time and validates it by ensuring that the number of oocysts arriving at the intake of the water treatment plant is not greater than the number discharged into the water channel.

Travel Distance Control for a Non-point Source in Overland Transport

As discussed previously, either free or floc- or particle-associated oocysts can be present in water. The ratio of free to floc-associated oocysts can be a function of various factors. The ratio used in this research (1:3) is taken from Medema et al. (1998), in the absence of other information. In
practice, this ratio will depend on soil type, the concentration of particulate organic matter, and different environmental conditions such as temperature, pH, suspended solids concentration, etc. Experimental determination of this ratio under a range of conditions would be important for detailed modelling.

Settling velocity is one of the most important physical properties of oocysts with respect to transport. Settling rate eventually limits both free and floc-associated oocyst travel distances. Because of the difference in settling rates, the travel distances of both free and floc-associated oocysts during runoff are different (Fig. 4) and should be calculated dynamically in each storm event. In the present model, it is assumed that oocysts start settling from the half-depth of runoff after detachment from a field. The estimated travel distances of both types of oocysts are taken into account in determining a line load of non-point oocysts to a water channel.

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**Fig. 3.** Cases for determining non-point source discharge time.

**Fig. 4.** The difference in travel distance for free and floc-associated oocysts.
Mathematical Description of the Model

Hypothetical Watershed

Due to the difficulty involved in identifying the non-point source function for oocyst generation in some cases and the fact that no analytical solution exists for some stages of oocyst transport, incorporating all mechanisms of the conceptual model into a simulation program is the most convenient way of investigating the relative importance of the parameters involved in oocyst transport. Several numerical techniques, such as finite difference or finite element methods, could be used. In this research, the finite difference form of the model was developed. Further details concerning the model, including the source code, can be found in Park (1999).

As discussed previously, the transport of free and floc-associated oocysts in some situations is different because of the difference in their settling velocities. Because of this difference, free and floc-associated oocyst transport modelling is handled separately in the following parts of the overall transport sequence: overland transport, lake sedimentation, and water channel transport upstream of a lake. In these components, oocyst settling velocities are important.

For transport purposes, the hypothetical watershed is divided into five sections: a point source upstream of a lake, the lake or reservoir itself, a stream channel from the outlet of the lake to a stream confluence, a tributary having a non-point source, and the stream section between the confluence and a water treatment plant intake. The criteria for defining the five sections are the different mechanisms involved in each section and relate mainly to water channel transport. Figure 5 shows the finite difference scheme for the five sections of the hypothetical watershed. The parameter settings used for the various sections in the finite difference scheme are given in Table 3.

The mathematical details of the finite difference schemes for each of the sections, including the appropriate boundary conditions, are given in Appendix A. Comments related to the mathematical form of the model and the finite difference scheme for each section are provided below.

Point Source Upstream of Lake (Section 1)

Free and floc-associated oocyst modelling are performed independently for this section because of the lake downstream and the importance of sedimentation in the lake. In this first section, the initial condition is set to zero in the river. In other words, it is assumed that there are no oocysts in the water before a rainfall event. Setting the initial condition to zero is standard practice for this type of modelling and simplifies the work. Because in reality there is a continuous point source discharge (the wastewater treatment plant) in Section 1, the early modelled oocyst concentrations in this section will be inaccurate, except for the case when a WWTP is being initially brought on stream.
Lake (Section 2)

As discussed previously, oocyst transport in a lake can be modelled to a first approximation by considering the lake either as a continuous stirred tank reactor (CSTR) or an ideal rectangular setting tank (IRST). Implementation of the numerical solutions for both a CSTR and an IRST is discussed in Appendix A. Because of the importance of the settling velocity for oocyst behaviour in a lake, free and floc-associated oocysts should be modelled separately.

The governing equation for a lake modelled as a CSTR was given previously (equation 8). The CSTR model is easily solved for certain simple loads. For the case when the loading is time dependent a simple numerical approach is adopted. Euler’s method is the simplest numerical method for solving ordinary differential equations. Appendix A shows the application of Euler’s method for solving the completely mixed lake model (Chapra 1997). The CSTR model can be solved with this method, even though the upstream oocyst source function may be very dynamic. This is one of the reasons for using a numerical approach for the model, because oocyst loadings may be difficult to represent accurately as pulse, step, linear, exponential, or sinusoidal functions.
An IRST model assumes ideal behaviour, as its name implies. The assumptions required for an ideal setting tank are discussed elsewhere (e.g., Reynolds 1982). The removal rates of both free and floc-associated oocysts are calculated by equation 13.

Thus, the concentration of oocysts leaving the lake is calculated from the oocyst input concentration multiplied by \((1-R_{oocyst})\). If the lake is dynamic with respect to parameters such as flow, oocyst concentration, etc., the IRST model will be poor at representing the system because of the limitations imposed by its assumptions.

### Between Lake and Stream Confluence (Section 3)

The third section of the hypothetical watershed is similar to the first section, the difference being that there is no source term in the third section. Thus, the only oocyst source is the oocyst concentration leaving the lake. However, in this section, there is no need for separate modelling of free and floc-associated oocysts, because the transport mechanisms are not closely related to oocyst settling velocity.

### Tributary with Non-point Source (Section 4)

In Section 4, there is a runoff-generated non-point source of oocysts that is converted to a line source at the water channel, as discussed by Park (1999). The difference in water channel transport between this section and Section 1 is that the source is non-point, and the source discharge is not continuous. Non-point source discharge duration is related to fac-

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**Table 3. Parameter settings for the finite difference method**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (m)</td>
<td>15,000</td>
<td>21,000 in Section 4</td>
</tr>
<tr>
<td>No. cells (nn)</td>
<td>10,000</td>
<td></td>
</tr>
<tr>
<td>Flow velocity (m/s)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Die-off coefficient (d(^{-1}))</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>Dispersion coefficient (cm(^{2}/s))</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>Distance step ((\Delta x)) (m)</td>
<td>1.5</td>
<td>2.1 in Section 4</td>
</tr>
<tr>
<td>Time step ((\Delta t)) (s)</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

\(^{a}\)Notes: The point source in Section 1 (cell 2) discharges continuously; the non-point source in Section 4 (cell 1429 is the first discharge cell) discharges intermittently as discussed in the text, and the WTP intake is located 10,000 m downstream from the stream confluence.

\(^{b}\)The same parameter setting is used in Sections 1, 3, 4 and 5 except as noted.
tors that affect overland oocyst transport, as described by Park (1999), and discussed later in this paper.

**Stream Confluence**

A stream confluence would be difficult to model in detail. In particular, if the flow in one stream is considerably greater than in the other, the mixing pattern would be very complicated. In the current model, the stream confluence is handled using a simple mass balance. As discussed previously, this approach adequately represents concentrations downstream of the confluence once complete mixing of the two streams has occurred. For the model in its present form, complete mixing of the two streams is assumed to occur instantaneously.

**Between Stream Confluence and WTP (Section 5)**

In this section only in-stream transport of oocysts occurs. As in Section 2, there is no source of oocysts in Section 5, however there is a water treatment plant intake. The oocyst concentration at the intake is the object of the modelling.

**Model Sensitivity Analysis**

At its present state of development, the model cannot be used to obtain quantitative results. However, it can be used to provide an indication of the relative importance of various factors for oocyst transport. This section addresses this by means of a sensitivity analysis.

**Approach**

Since a data set for the model was not available, required values were either obtained from the literature or assigned reasonable values in cases where no data were available or where a number of different values could be used, such as the location of the manure-applied area. It is important that the values assumed for the base setting are reasonable, because in a complex model one factor may be critical when other factors are within a certain range. The assumed factors were included in the sensitivity analysis to evaluate their importance. Table 4 shows the base settings for the parameters.

Although the hypothetical watershed contains both point and non-point sources, their relative impact on the oocyst concentration at the drinking water intake in Section 5 depends on the assumed sizes of the two sources. Therefore, two sensitivity analyses are performed: a point source analysis (involving Sections 1, 2, 3 and 5 of the hypothetical watershed) and a non-point source analysis (involving Sections 4 and 5).

Due to the different units of the various model parameters, their relative impact on the state variable (oocyst concentration) can only be established using a normalized sensitivity analysis. The basic equation for normalization is given by
\[ S_n = \frac{\partial \phi}{\partial \alpha_i} \cdot \frac{\alpha_i}{\phi} \] (15)

where \( S_n \) is the normalized sensitivity coefficient, \( \phi \) is the state variable, and \( \alpha_i \) is the model parameter being investigated. For the \( \frac{\partial \phi}{\partial \alpha_i} \) term, a first-backwards approximation or the central difference form can be used. In this analysis, the central difference form was used.

The central difference form is given by:

\[ \frac{\partial \phi}{\partial \alpha_i} \approx \frac{\phi(\alpha_i + \Delta \alpha_i) - \phi(\alpha_i - \Delta \alpha_i)}{2\Delta \alpha_i} \] (16)

where \( \Delta \alpha_i \) is the difference in a perturbed parameter.

A positive sign indicates that an increase in \( \alpha_i \) will result in an increase in \( \phi \) (the state variable). A \( \Delta \) value of 1% and was used for the analysis. Park (1999) also used a \( \Delta \) value of 2%, and found that the trends were the same as at 1%.

Results

Point source oocysts

The results for the normalized sensitivity analysis for point source oocysts are shown in Fig. 6. It will be recalled that in the hypothetical watershed a lake is located downstream of the point source. From Fig. 6, it can be seen that the most important factors are the concentration of oocysts in the point source discharge (Opoint1), and the human population contributing to that discharge (popu1). As would be expected, both of these factors are of equal importance, because each has the same impact on numbers of oocysts discharged. For the base settings of the parameters used in this sensitivity analysis, an equal percentage change in either the oocyst die-off rate (dieoff1) or the lake retention time (retime1), has a much lower impact on oocyst concentration.

Non-point source oocysts

In the model, non-point source oocysts are generated by runoff. Therefore, the parameters examined in the sensitivity analysis are related to the original source of the oocysts, the detachment process and runoff itself. The results (Fig. 7) show that the most critical factor is the amount of rainfall.

The next section discusses the results of the sensitivity analysis for point and non-point sources in further detail.
Table 4. Base settings used in the model sensitivity analysis.

<table>
<thead>
<tr>
<th>Parameter valuea</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Point source</strong></td>
<td></td>
</tr>
<tr>
<td>Oocyst concentration</td>
<td>5 oocyst/L Based on Rose (1988)</td>
</tr>
<tr>
<td>Population</td>
<td>80,000 people Approximate population of City of Waterloo</td>
</tr>
<tr>
<td>Pipe diameter</td>
<td>1.5 m Assumed</td>
</tr>
<tr>
<td><strong>Advection – dispersion equation</strong></td>
<td>Contaminant transport in surface water</td>
</tr>
<tr>
<td>D (dispersion coefficient)</td>
<td>1000 cm²/s</td>
</tr>
<tr>
<td>k (decay rate coefficient)</td>
<td>0.012 oocysts/day Robertson et al. (1992)</td>
</tr>
<tr>
<td>River velocity</td>
<td>1 m/s High flow in the Grand River</td>
</tr>
<tr>
<td><strong>Water channel</strong></td>
<td>Assumed</td>
</tr>
<tr>
<td>Cross-sectional area of channel</td>
<td>200 m²</td>
</tr>
<tr>
<td>Ratio of free to floc-associated oocysts</td>
<td>1:3 Medema et al. (1998)</td>
</tr>
<tr>
<td>Detachment and any loss</td>
<td>10% Assumed</td>
</tr>
<tr>
<td><strong>Non-point source</strong></td>
<td>Zhang’s (1999) estimation based on Hansen and Ongerth (1991)</td>
</tr>
<tr>
<td>Number of oocysts on a field</td>
<td>4.3 × 10¹⁰</td>
</tr>
<tr>
<td><strong>Runoff</strong></td>
<td>Assumed</td>
</tr>
<tr>
<td>Precipitation</td>
<td>1 inch (25 mm)</td>
</tr>
<tr>
<td>AMC</td>
<td>II</td>
</tr>
<tr>
<td>Land use</td>
<td>Pasture or range, good condition Assumed</td>
</tr>
<tr>
<td>CN (curve number)</td>
<td>80 Calculated or read off from the graph – SCS (1972)</td>
</tr>
<tr>
<td><strong>Manning’s equation</strong></td>
<td>Taken from land use</td>
</tr>
<tr>
<td>n (roughness coefficient)</td>
<td>0.03</td>
</tr>
<tr>
<td>Slope</td>
<td>0.02 Assumed</td>
</tr>
<tr>
<td>Runoff time</td>
<td>1 hour Assumed</td>
</tr>
<tr>
<td><strong>Overland flow</strong></td>
<td>Assumed</td>
</tr>
<tr>
<td>Size of manure-applied area</td>
<td>500 m × 500 m</td>
</tr>
<tr>
<td>Distance from water channel</td>
<td>100 m</td>
</tr>
</tbody>
</table>

a Values obtained from the literature or assumed.
Discussion

An important aspect of the conceptual model is the division of oocysts into free and floc-associated. This leads to a difference in settling velocities, as discussed previously, which has implications for the over-land transports of oocysts. The approach to transport by runoff in the current model is different from that of Walker et al. (1999). In their fate and transport model for Cryptosporidium, they assumed that oocysts and
manure moved together. The model described in the present paper leads to greater oocyst transmission to a watercourse because free oocysts detached into water can move along with overland flow. This means that runoff water with very low turbidity can carry oocysts to a watercourse. This transmission can occur because of the relatively small size of oocysts and the fact that their density is close to that of water (1.05 g/cm³ [Medema et al. 1998] versus an average of about 2.7 g/cm³ for clay sediment). The importance of the factors related to overland transport is an additional reason why the sensitivity analysis for point and non-point source oocysts needs to be performed separately.

**Point source oocysts**

The order of importance of factors for point source oocyst transport as estimated by normalized sensitivity analysis (Fig. 6) was as follows:

Oocyst concentration = population >> lake retention time = die-off

For a point source oocyst discharge (i.e., a wastewater treatment plant), the number of oocysts discharged per unit time is proportional to both the human population served by the treatment plant and to the oocyst concentration (number per unit volume) in the discharge. Thus, the normalized sensitivity coefficients of the two factors are the same. This result would be essentially the same if the point source were a combined sewer outfall. Since the population served by a WWTP cannot normally be reduced, the sensitivity analysis suggests, for situations where the parameter values are similar to the base settings used herein, that point source contributions to oocyst levels at a water treatment plant intake could be most effectively mitigated by maximizing the removal or inactivation of oocysts in upstream wastewater treatment plants. This would normally imply a wastewater disinfection step capable of oocyst inactivation, and would be an argument for the use of a technology such as UV instead of chlorination for wastewater disinfection.

Normally, the retention time of a lake may vary from several days to more than a year. In the sample watershed, the retention time of the lake is 4.3 days. Oocyst removal by sedimentation was estimated at 5.3% and total oocyst removal, which includes die-off during the residence time, was 10.1%. If the length and width of the lake are each increased to four times the original, the residence time becomes about 70 days. In this case, 57% of the incoming oocysts would be removed by sedimentation and the total oocyst removal with die-off becomes 81%, which represents a dramatic increase. Even though lake residence time and the die-off rate were not the most significant factors for point source oocyst transport based on the normalized sensitivity analysis, lakes or reservoirs with a long residence time should act as a barrier that substantially reduces numbers of oocysts arriving at downstream water treatment plants.

Walker et al. (1999) performed *Cryptosporidium* modelling for the New York City water supply system, which is dominated by several large
reservoirs with residence times on the order of many months to years. Before the present conceptual model could be applied to such watersheds, the submodels for oocyst fate and transport in lakes would need to be more fully developed because of the dominant role that such water bodies play in systems such as the one for New York City.

**Non-point source oocysts**

Figure 7 ranked the relative importance of various factors affecting in-stream oocyst concentrations derived from the non-point source as follows:

Amount of rainfall (rainfall1) > number of oocysts on a field (Onon1) = detachment ratio to runoff (Rde1) = river velocity (river_v1) > land slope (slope1) > distance to watercourse (distance1).

This order could vary somewhat if different base settings of the parameters were used. For example, for the given storm event, no oocysts would reach the watercourse from the manure-applied area if its distance from the watercourse were increased by more than 30%.

Because the travel distance of free and floc-associated oocysts in overland flow is different, only free oocysts can reach the watercourse when the distance from the manure-applied area is above a certain value. Thus, the distance parameter functions as a switch. It turns on or off the oocyst discharge (free, or both types or neither) depending on the distance for a given intensity of storm and given physical conditions such as land slope.

Figure 7 indicates that the most critical factor for non-point source oocyst transport is the amount of rainfall, which is of course beyond human control. The second most important factors are the number of oocysts on a field, the detachment ratio and the river velocity. Controlling the river velocity is normally not practical. Similarly, land slope is essentially a noncontrollable factor. However, oocyst generation from a field can be controlled by modifying farming practices. For example, the detachment ratio used assumes that oocysts are exposed on top of the soil. By applying manure in the soil, this detachment ratio can be dramatically reduced. Applying manure in areas away from a watercourse is also a way to minimize the number of non-point source oocysts reaching that watercourse. Theoretically, runoff can be reduced by applying manure to areas with different antecedent land use, but this may be difficult in practice.

**Summary**

This paper describes the formulation of a model for *Cryptosporidium* oocyst transport in watersheds. The ultimate objective of the modelling is to provide order-of-magnitude estimates of oocyst concentrations at the intake of a water treatment plant. Estimates to this level of accuracy are useful and appropriate for treatment plant design. The model includes the
generation of oocysts from both point and non-point sources, the over-
land transport of oocysts from a non-point source to a watercourse, in-
stream oocyst transport and transport through a lake or reservoir. The
model also incorporates the confluence of two streams, and includes a
function for oocyst die-off.

The finite difference scheme of the model was developed for five sec-
tions in a hypothetical watershed: 1. A point source (WWTP discharge); 2.
A lake or reservoir, which can be modelled either as a continuous stirred
tank reactor, or as an ideal rectangular sampling tank; 3. The section of
stream channel leading from the outlet of the lake or reservoir to the con-
fluence with another stream; 4. A tributary stream with a non-point
source; and 5. The stream section from the confluence to a water treatment
plant intake.

The criteria for defining each of the five sections were the different
transport mechanisms involved. The stream confluence is handled by a
simple mass and flow balance. A key element and contribution of the
model is the division of oocysts into two categories: free and floc-associ-
ated. This distinction is important with regard to overland transport.

Data to test the model are not currently available. Therefore the rela-
tive importance of the various factors was tested for the hypothetical
watershed. Reasonable base settings for each parameter were used and a
normalized sensitivity analysis (in which each factor was varied by the
same percentage amount) was performed. Separate sensitivity analyses
were performed for the point and non-point sources.

For the point source, the most important factors were the concentra-
tion of oocysts in the discharge and population contributing to that dis-
charge. For the non-point source, the most important factor was the
amount of rainfall. There were differences in importance among the other
factors, however their relative importance is likely to be situation-specif-
ic. A practical conclusion that can be drawn from the sensitivity analysis
is that wastewater disinfection capable of effectively inactivating oocysts
may be important.

The major contribution of the model is in the definition of the mech-
anisms involved in oocyst transport within a watershed. At its present
stage of development, the model cannot be used to obtain quantitative
results. However, it does give insights into the relative importance of var-
ious factors. It can also be applied to other pathogens such as Giardia. It
provides a framework for data collection and identifies areas where
experimental investigations are required to avoid or minimize the need
for simplifying assumptions. The model provides a base for further mod-
elling. Ultimately, further more detailed modelling could be expected to
help development of management practices to minimize the contribu-
tions of both point and non-point sources to pathogen loadings at drink-
ing water intakes.
Acknowledgements

We acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada and the assistance of Angela Semple, Ada Chavez and Sarah Dorner in completion of the manuscript.

References


Appendix A

The Finite Difference Scheme of the Model

Water Channel Transport

The advection-dispersion equation, which forms the basis for modeling transport in the water channel, is a second-order differential equation, requiring two boundary conditions and one initial condition for solution. Equation A1 (equivalent to equation 7 in the main text) is the classical advection-dispersion equation, including a die-off term:

$$\frac{\partial N}{\partial t} = D \frac{\partial^2 N}{\partial x^2} - \nu \frac{\partial N}{\partial x} - kN + S(x, t) \quad 0 < x < L$$  \hspace{1cm} (A1)

where $N$ is the oocyst concentration (oocyst/m$^3$), $D$ is the dispersion coefficient (m$^2$/s), $\nu$ is the river velocity (m/s), $k$ is a first order decay coefficient (s$^{-1}$), and $S$ is a source function.

Figure A1 shows the basic finite difference scheme for the water channel, using central block difference. The initial condition is $N(x,0) = 0$ and the two boundary conditions are $N(0,t) = 0$ at $x = 0$ and $\frac{\partial N}{\partial x} = 0$ at $x = L$. The first is a type I boundary condition, while the second is a type II boundary condition. The latter boundary condition cannot be proven but is considered reasonable under the circumstances. The finite difference method for pointwise expressions is given by:

$$\frac{\partial N}{\partial t} \approx \frac{N_{i}^{k+1} - N_{i}^{k}}{\Delta t}, \text{ where } k = \text{time step}$$ \hspace{1cm} (A2)

$$\frac{\partial^2 N}{\partial x^2} \approx \frac{N_{i+1} - 2N_{i} + N_{i-1}}{\Delta x^2}$$ \hspace{1cm} (A3)

$$\frac{\partial N}{\partial x} \approx \frac{N_{i+1} - N_{i-1}}{2\Delta x}$$ \hspace{1cm} (A4)

Fig. A1. The basic finite difference scheme for a water channel.
Therefore, the solution for the advection-dispersion equation (equation A1) can be determined using the fully implicit method, given by:

\[
\frac{N_i^{k+1} - N_i^k}{\Delta t} = D\left(\frac{N_i^{k+1} - 2N_i^k + N_i^{k-1}}{\Delta x^2}\right) - v\left(\frac{N_i^{k+1} - N_i^k}{2\Delta x}\right) - kN_i^{k+1} + S_i^k \tag{A5}
\]

Rearranging equation A5 gives:

\[
a_1 N_{i-1}^{k+1} + a_2 N_i^{k+1} + a_3 N_{i+1}^{k+1} = \frac{N_i^k}{\Delta t} + S_i^k,
\]

where

\[
a_1 = -\frac{2D + \Delta x \cdot v}{2\Delta x^2},
\]

\[
a_2 = \frac{\Delta x^2 + 2\Delta t \cdot D + k\Delta t \Delta x^2}{\Delta x^2 \Delta t},
\]

\[
a_3 = \frac{\Delta x \cdot v - 2D}{2\Delta x^2},
\]

The boundary condition \(N(0,t) = 0\) at \(x = 0\) (type I) can be replaced by \(N_1^{k+1} = 0\), and \(\frac{\partial N}{\partial x} = 0\) at \(x = L\) (type II) can be replaced by:

\[
\left.\frac{\partial N}{\partial x}\right|_{x=L} \approx \frac{N_{n+1} - N_{n-1}}{2\Delta x} = 0 \Rightarrow N_{n+1} = N_{n-1} \tag{A7}
\]

Therefore, the row entry for block \(n\) is:

\[
(a_1 + a_3) N_{n-1}^{k+1} + a_2 N_n^{k+1} = \frac{N_n^k}{\Delta t} + S_n^k \tag{A8}
\]

To solve this row entry for a block, a tridiagonal matrix is generated by:

\[
\begin{bmatrix}
1 & 0 & \cdots \\
(a_1 + a_3) & a_2 & a_3 \\
\vdots & a_1 & a_2 & a_3 \\
\vdots & \ddots & \ddots & \ddots \\
\end{bmatrix}
\begin{bmatrix}
N_1 \\
N_2 \\
\vdots \\
N_n
\end{bmatrix} =
\begin{bmatrix}
0 + S_1^k \\
N_2^k + S_2^k \\
\vdots \\
N_n^{k+1} + S_n^k
\end{bmatrix} \tag{A9}
\]
The Thomas algorithm written in C (William et al. 1992) was used (Park 1999) to solve this tridiagonal matrix in each water channel.

**Point Source Upstream of Lake (Section 1)**

Figure A2 shows Section 1 as a source term with boundary conditions. The actual mass flux at \( x = L \) is given by \( \nu N - D \frac{\partial N}{\partial x} \) at \( x = L \). Because the mass flux due to advection (\( \nu N \)) is much greater than that due to dispersion, the approximation \( \frac{\partial N}{\partial x} = 0 \) is justified.

**Lake (Section 2)**

The following equations show the application of Euler’s method for solving the completely mixed lake model (Chapra 1997):

\[
\frac{dN}{dt} = \frac{W(t)}{V} - LN
\]  

\[\text{where} \quad S(t) = \frac{W(t)}{V} \]  

\[1 = \frac{Q}{V} + k + \frac{\nu_s}{H_{\text{lake}}} \]

For a numerical solution, difference approximations can be used to express derivatives in arithmetic terms. For example, using a forward difference, approximation of the first derivative of \( N \) with respect to \( t \) is given by:

\[
\frac{dN_i}{dt} \cdot \frac{DN}{Dt} = \frac{N_{i+1} - N_i}{t_{i+1} - t_i}
\]  

**Fig. A2.** The finite difference scheme for Section 1.

Boundary conditions: \( N(0,t)=0 \) at \( x = 0 \) and \( \frac{\partial N}{\partial x} = 0 \) at \( x = L \).
where $N_i$ and $N_{i+1}$ are concentrations at a present and a future time $t_i$ and $t_{i+1}$, respectively. Substituting equation A13 into equation A10 yields:

$$\frac{N_{i+1} - N_i}{t_{i+1} - t_i} = \frac{W(t)}{V} - \lambda N_i$$

which can be solved for:

$$N_{i+1} = N_i + \left[ \frac{W(t)}{V} - \lambda N_i \right] (t_{i+1} - t_i)$$

**Between Lake and Stream Confluence (Section 3)**

The finite difference scheme used for this section is shown in Fig. A3.

**Tributary with Non-point Source (Section 4)**

The finite difference scheme for this section is given in Fig. A4. The non-point source has been converted to a line source at the water channel.

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**Fig. A3.** The finite difference scheme for Section 3.

**Fig. A4.** The finite difference scheme for Section 4.
Stream Confluence

Based on the advective mass balance (Fig. A5), oocyst flux at the stream confluence is given by:

\[ N_3 Q_3 + N_4 Q_4 = N_5 Q_5 \]  \hspace{1cm} (A16)

and water flow is given by:

\[ Q_3 + Q_4 = Q_5 \]  \hspace{1cm} (A17)

where \( N_3 \) is the oocyst concentration in the last cell of Section 3, \( Q_3 \) is the flow in Section 3, \( N_4 \) is the oocyst concentration in the last cell of Section 4, \( Q_4 \) is the flow in Section 4, \( N_5 \) is the oocyst concentration in the first cell of Section 5, and \( Q_5 \) is the flow in Section 5.

Considering oocyst flux at the stream confluence, one boundary condition for Section 5 is of type III based on mass balance, and is given by:

\[ Q_5 N_5 = -\left[ Q_5 N - DA \frac{\partial N}{\partial x} \right] \]  \hspace{1cm} (A18)

where \( N \) is the concentration in the first cell of Section 5.

The sign of the right hand side is determined by running the model and checking the mass balance. For example, if equation A18 gives negative values of oocyst concentration in the fifth section, the negative sign should be switched to a positive sign. In this way, the boundary condition at the stream confluence can be determined.

Fig. A5. Mass balance for stream confluence.
Between Stream Confluence and WTP (Section 5)

One of the boundary conditions for Section 5 was discussed immediately above with respect to the stream confluence. The other boundary condition is type II as used previously. The oocyst concentration at the water treatment plant intake in Section 5 is the object of the modelling. Details of the finite difference scheme for Section 5 are shown in Fig. A6.

![Finite Difference Scheme](image)

**Fig. A6.** The finite difference scheme for Section 5.

Boundary conditions:

\[
Q_s N_s = \left[ Q_s N - DA \frac{\partial N}{\partial x} \right] \text{ at } x = 0 \text{ and } \frac{dN}{dx} = 0 \text{ at } x = L.
\]